Modeling Large-Scale Atmospheric and Oceanic Flows 2

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Importance of moisture in the atmosphere: obvious. Influences large-scale dynamics via the latent heat release, due to condensation and precipitation.

Atmospheric circulation modeling: equation of state of the moist air extremely complex. Discretization/averaging: problematic.

Current parametrizations of precipitations and latent heat release:

relaxation to the equilibrium (saturation) profile of humidity ⇒ threshold effect ⇒ essential nonlinearity Consequences: no linear limit; linear thinking: modal decomposition, linear stability analysis, etc impossible ⇒ problems in quantifying predictability of moist - convective dynamical systems.

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Understanding the influence of condensation and latent heat release upon large-scale dynamical processes

Reminder:

- Simplest model for large-scale motions: rotating shallow water.
- Link with primitive equations: vertical averaging
- Baroclinic effects: 2 (or more) layers.

Problem with this approach for moist air: averaging of essentially nonlinear equation of state.

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Combine (standard) vertical averaging of primitive equations between the isobaric surfaces with that of Lagrangian conservation of moist enthalpy

- Allow for convective fluxes (extra vertical velocity) across the isobars
- Link these fluxes to condensation
- Use relaxation parametrization in terms of bulk moisture in the layer for the condensation/precipitation

Methodology



Advantages:

- Simplicity, qualitative analysis of basic phenomena straightforward
- Fully nonlinear in the hydrodynamic sector
- Well-adapted for studying discontinuities, in particular precipitation fronts
- Efficient numerical tools available (finite-volume codes for shallow water)
- Various limits giving known models
- Inclusion of topography (gentle or steep) straightforward

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Primitive equations in pseudo-height coordinates

$$egin{aligned} & rac{d}{dt} oldsymbol{v} + f oldsymbol{k} imes oldsymbol{v} & = -
abla \phi \end{aligned} \ & rac{d}{dt} heta = 0 \ &
abla \cdot oldsymbol{v} + \partial_z w = 0 \ &
abla_z \phi = g rac{ heta}{ heta_0} \end{aligned}$$

 $\mathbf{v} = (u, v)$ and w - horizontal and vertical velocities, $\frac{d}{dt} = \partial_t + \mathbf{v} \cdot \nabla + w \partial_z$, f - Coriolis parameter, θ - potential temperature, ϕ - geopotential.

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Condensation turned off: conservation of specific humidity of the air parcel:

$$\frac{d}{dt}q=0.$$

Condensation turned on: θ and q equations acquire source and sink. Yet the moist enthalpy $\theta + \frac{L}{c_p}q$, where L-latent heat release, c_p -specific heat, is conserved for any air parcel on isobaric surfaces:

$$\frac{\textit{d}}{\textit{d}t}\left(\theta+\frac{\textit{L}}{\textit{c}_{p}}\textit{q}\right)=0,$$

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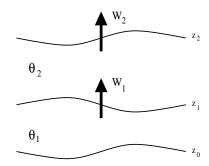
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Vertical averaging with convective fluxes

3 material surfaces:

$$w_0 = \frac{dz_0}{dt}, \quad w_1 = \frac{dz_1}{dt} + W_1, \quad w_2 = \frac{dz_2}{dt} + W_2.$$



Mean-field + constant mean $\theta \rightarrow$

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$\begin{cases} \begin{array}{l} \partial_t \textbf{\textit{v}}_1 + (\textbf{\textit{v}}_1 \cdot \nabla) \textbf{\textit{v}}_1 + f \textbf{\textit{k}} \times \textbf{\textit{v}}_1 = -\nabla \phi(\textbf{\textit{z}}_1) + g \frac{\theta_1}{\theta_0} \nabla \textbf{\textit{z}}_1, \\ \partial_t \textbf{\textit{v}}_2 + (\textbf{\textit{v}}_2 \cdot \nabla) \textbf{\textit{v}}_2 + f \textbf{\textit{k}} \times \textbf{\textit{v}}_2 = -\nabla \phi(\textbf{\textit{z}}_2) + g \frac{\theta_2}{\theta_0} \nabla \textbf{\textit{z}}_2 + \frac{\textbf{\textit{v}}_1 - \textbf{\textit{v}}_2}{h_2} & \textbf{\textit{W}}_1 \text{ valor laws} \\ & \textbf{\textit{W}}_2 \text{ valor laws} & \textbf{\textit{v}}_2 = -\nabla \phi(\textbf{\textit{z}}_2) + g \frac{\theta_2}{\theta_0} \nabla \textbf{\textit{z}}_2 + \frac{\textbf{\textit{v}}_1 - \textbf{\textit{v}}_2}{h_2} & \textbf{\textit{W}}_2 \text{ valor laws} \\ & \textbf{\textit{v}}_3 = -\nabla \phi(\textbf{\textit{v}}_3) + g \frac{\theta_2}{\theta_0} \nabla \textbf{\textit{v}}_3 + g \frac{\theta_3}{\theta_0} \nabla \textbf{\textit{v}}_3 + g$ $\begin{cases} \partial_t h_1 + \nabla \cdot (h_1 \mathbf{v}_1) = -W_1, \\ \partial_t h_2 + \nabla \cdot (h_2 \mathbf{v}_2) = +W_1 - W_2, \end{cases}$

Linking convective fluxes to precipitation I

Bulk humidity: $Q_i = \int_{z_{i-1}}^{z_i} q dz$. Precipitation sink:

$$\partial_t Q_i + \nabla \cdot (Q_i \mathbf{v}_i) = -P_i.$$

In precipitating regions ($P_i > 0$), moisture is saturated $q(z_i) = q^s(z_i)$ and the temperature of the air-mass Widtdxdy convected due to the latent heat release $\theta(z_i) + \frac{L}{c_s} q^s(z_i)$, is the one of the upper layer: θ_{i+1} . We assume "dry" stable background stratification:

$$\theta_{i+1} = \theta(z_i) + \frac{L}{c_p}q(z_i) \approx \theta_i + \frac{L}{c_p}q(z_i) > \theta_i,$$

with constant $\theta(z_i)$ and $q(z_i)$.

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$$W_i = \beta_i P_i$$

with a positive-definite coefficient

$$eta_i = rac{L}{c_p(heta_{i+1} - heta_i)} pprox rac{1}{q(z_i)} > 0.$$

Last step: relaxation formula with relaxation time τ .

$$P_i = \frac{Q_i - Q_i^s}{\tau} H(Q_i - Q_i^s)$$

where H(.) is the Heaviside (step) function.

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$$\begin{cases} \partial_t \mathbf{v}_1 + (\mathbf{v}_1 \cdot \nabla) \mathbf{v}_1 + f \mathbf{k} \times \mathbf{v}_1 = -g \nabla (h_1 + h_2), \\ \partial_t \mathbf{v}_2 + (\mathbf{v}_2 \cdot \nabla) \mathbf{v}_2 + f \mathbf{k} \times \mathbf{v}_2 = -g \nabla (h_1 + \alpha h_2) + \frac{\mathbf{v}_1 - \mathbf{v}_2}{h_2} \beta P, \\ \partial_t h_1 + \nabla \cdot (h_1 \mathbf{v}_1) = -\beta P, \\ \partial_t h_2 + \nabla \cdot (h_2 \mathbf{v}_2) = +\beta P, \\ \partial_t Q + \nabla \cdot (Q \mathbf{v}_1) = -P, \quad P = \frac{Q - Q^s}{\tau} H(Q - Q^s) \end{cases}$$

 $\alpha = \frac{\theta_2}{\theta_L}$ - stratification parameter.

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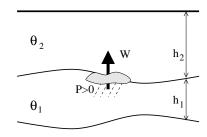
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Sketch of the model



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Immediate relaxation limit

$$au o 0, \; \Rightarrow \; P = -Q^s
abla \cdot \mathbf{v}_1 \; (\text{Gill, 1982}), \; \text{and}$$
 $\partial_t \mathbf{v}_1 + (\mathbf{v}_1 \cdot
abla) \mathbf{v}_1 + f \mathbf{k} \times \mathbf{v}_1 = -g
abla (h_1 + h_2),$ $\partial_t \mathbf{v}_2 + (\mathbf{v}_2 \cdot
abla) \mathbf{v}_2 + f \mathbf{k} \times \mathbf{v}_2 = -g
abla (h_1 + \alpha h_2)$ $-\frac{\mathbf{v}_1 - \mathbf{v}_2}{h_2} \beta Q^s
abla \cdot \mathbf{v}_1,$ $\partial_t h_1 +
abla \cdot (h_1 \mathbf{v}_1) = +\beta Q^s
abla \cdot \mathbf{v}_1,$ $\partial_t h_2 +
abla \cdot (h_2 \mathbf{v}_2) = -\beta Q^s
abla \cdot \mathbf{v}_1,$

humidity staying at the saturation value: $Q = Q^s$.

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$$\mathbf{v}^{bt} = \frac{h_1 \mathbf{v}_1 + h_2 \mathbf{v}_2}{h_1 + h_2}, \ \mathbf{v}^{bc} = \mathbf{v}_1 - \mathbf{v}_2,$$

and linearizing in the hydrodynamic sector gives:

$$\left\{ \begin{array}{l} \partial_t \mathbf{v}^{bc} + f \mathbf{k} \times \mathbf{v}^{bc} = -g_e \nabla \eta, \\ \partial_t \eta + H_e \nabla \cdot \mathbf{v}^{bc} = -\beta P, \\ \partial_t Q + Q_e \nabla \cdot \mathbf{v}^{bc} = -P, \end{array} \right. ,$$

where $g_e = g(\alpha - 1)$, $Q_e = \frac{H_e}{H_1}Q^s$, η - perturbation of the interface, H_e - equivalent height.

Model first proposed by Gill (1982) and studied by Majda et al (2004, 2006, 2008).

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$$\frac{d_1^{(0)}}{dt}(\nabla^2\psi_1 + y - \frac{\eta_1}{D_1}) = \frac{\beta P}{D_1},
\frac{d_2^{(0)}}{dt}(\nabla^2\psi_2 + y - \frac{\eta_2}{D_2}) = -\frac{\beta P}{D_2},$$

Here $\frac{d_i^{(0)}}{dt} = \partial_t + (\boldsymbol{v}_i^{(0)} \cdot \nabla)$, $\boldsymbol{k} \times \boldsymbol{v}_i^{(0)} = -\nabla \psi_i$, $D_i = \frac{H_i}{H_0}$, and $\psi_{1,2}$ (geostrophic streamfunctions) are related to the free-surface (η_2) and interface (η_1) perturbations as:

$$\psi_1 = \eta_1 + \eta_2, \quad \psi_2 = \eta_1 + \alpha \eta_2.$$

Limiting equations and relation to the known models

In the limit $H_1/(H_1+H_2) \rightarrow 0$ the reduced-gravity one-layer moist-convective shallow water follows (Bouchut, Lambaerts, Lapeyre & Zeitlin, 2009):

$$\begin{cases} \partial_t \mathbf{v}_1 + (\mathbf{v}_1 \cdot \nabla) \mathbf{v}_1 + f \mathbf{k} \times \mathbf{v}_1 = -\nabla \eta, \\ \partial_t \eta + \nabla \cdot \{\mathbf{v}_1 (1 + \eta)\} = -\beta \mathbf{P}, \\ \partial_t Q + \nabla \cdot (Q \mathbf{v}_1) = -\mathbf{P}, \end{cases}$$

(Nondimensional equations, η - free-surface perturbation)

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Horizontal momentum

$$egin{aligned} (\partial_t + oldsymbol{v}_1 \cdot
abla) (oldsymbol{v}_1 h_1) + oldsymbol{v}_1 h_1
abla \cdot oldsymbol{v}_1 + f oldsymbol{k} imes (oldsymbol{v}_1 h_1) \ &= -g
abla rac{h_1^2}{2} - g h_1
abla h_2 - oldsymbol{v}_1 oldsymbol{eta} oldsymbol{P}, \ (\partial_t + oldsymbol{v}_2 \cdot
abla) (oldsymbol{v}_2 h_2) + oldsymbol{v}_2 h_2
abla \cdot oldsymbol{v}_2 + f oldsymbol{k} imes (oldsymbol{v}_2 h_2) \end{aligned}$$

$$=-lpha g
ablarac{h_2^2}{2}-gh_2
abla h_1+oldsymbol{v_1}oldsymbol{eta P},$$

Red: moist convection drag. Total momentum: $\mathbf{v}_1 h_1 + \mathbf{v}_2 h_2$ is not affected by convection.

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$$\begin{cases} e_1 = h_1 \frac{\mathbf{v}_1^2}{2} + g \frac{h_1^2}{2}, \\ e_2 = h_2 \frac{\mathbf{v}_2^2}{2} + g h_1 h_2 + \alpha g \frac{h_2^2}{2}, \end{cases}$$

For the total energy $E = \int dx dy (e_1 + e_2)$ we get:

$$\partial_t \mathbf{E} = -\int d\mathbf{x} \, \beta P\left(gh_2(1-\alpha) + \frac{(\mathbf{v}_1 - \mathbf{v}_2)^2}{2}\right).$$

1st term: production of PE (for stable stratification); 2nd term destruction of KE.

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$$(\partial_t + \mathbf{v}_1 \cdot \nabla) \frac{\zeta_1 + f}{h_1} = \frac{\zeta_1 + f}{h_1^2} \beta P,$$

$$(\partial_t + \mathbf{v}_2 \cdot \nabla) \frac{\zeta_2 + f}{h_2} = -\frac{\zeta_2 + f}{h_2^2} \beta P + \frac{\mathbf{k}}{h_2} \cdot \left\{ \nabla \times \left(\frac{\mathbf{v}_1 - \mathbf{v}_2}{h_2} \beta P \right) \right\}$$

where
$$\zeta_i = \mathbf{k} \cdot (\nabla \times \mathbf{v}_i) = \partial_x v_i - \partial_y u_i$$
 ($i = 1, 2$)- relative vorticity.

PV in each layer is not a Lagrangian invariant in precipitating regions.

Moist enthalpy and moist PV

Moist enthalpy in the lower layer: $m_1 = h_1 - \beta Q$ and is always locally conserved:

$$\partial_t m_1 + \nabla \cdot (m_1 \mathbf{v}_1) = 0.$$

Conservation of the moist enthalpy in the lower layer allows to derive a new Lagrangian invariant, the moist PV:

$$(\partial_t + \boldsymbol{v}_1 \cdot \nabla) \frac{\zeta_1 + f}{m_1} = 0.$$

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1-d reduction: $\partial_{y}(...) = 0$, \Rightarrow quasilinear system:

$$\partial_t \mathbf{f} + \mathbf{A}(\mathbf{f})\partial_X \mathbf{f} = \mathbf{b}(\mathbf{f}).$$

Characteristic equation: $det(\mathbf{A} - c\mathbf{I}) = 0$

"Dry" characteristic equation

$$\mathcal{F}(c) = \left\{ (u_1 - c)^2 - gh_1 \right\} \left\{ (u_2 - c)^2 - \alpha gh_2 \right\} - gh_1 gh_2$$

▶ "Moist" characteristic equation $(\tau \rightarrow 0)$

$$\mathcal{F}^{m}(c) = \mathcal{F}(c) + ((u_1 - u_2)^2 - (\alpha - 1)gh_2)g\beta Q^s = 0.$$

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▶ "Dry" characteristics:

$$C_{\pm} = g(H_1 + \alpha H_2) \frac{1 \pm \sqrt{\Delta}}{2},$$

"Moist" characteristics:

$$C_{\pm}^{m}=g(H_{1}+\alpha H_{2})\frac{1\pm\sqrt{\Delta^{m}}}{2}.$$

Here $C = c^2$ and

$$\Delta = 1 - \frac{4H_1H_2(\alpha - 1)}{(H_1 + \alpha H_2)^2} = \frac{(H_1 - \alpha H_2)^2 + 4H_1H_2}{(H_1 + \alpha H_2)^2}$$
$$\Delta^m = \Delta + \frac{4(\alpha - 1)\beta Q^s H_2}{(H_1 + \alpha H_2)^2}.$$

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Moist vs dry characteristic velocities

 c^m is real for positive moist enthalpy of the lower layer in the state of rest : $M_1 = H_1 - \beta Q^s > 0$, and

$$C_{-}^{m} < C_{-} < \frac{g(H_{1} + \alpha H_{2})}{2} < C_{+} < C_{+}^{m},$$

for $0 < M_1 < H_1 \Rightarrow$ moist internal (mainly baroclinic) mode propagates slower than the dry one, consistent with observations.

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Rankine-Hugoniot (RH) conditions (immediate relaxation):

$$\begin{cases} -s[v_1h_1 + v_2h_2] + [u_1v_1h_1 + u_2v_2h_2] = 0, \\ -s[m_1] + [m_1u_1] = 0, \\ -s[h_2] + [h_2u_2 + \beta Q^su_1] = 0. \end{cases}$$

s - propagation speed of the discontinuity. Remark: mass conservation \to moist enthalpy conservation in the lower layer.

Due to $\lim_{x_s \to a} \lim_{b \to x_s} \int_a^b P = 0$, P does not enter RH conditions for u, v, h.

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RH conditions linearized about the rest state:

$$\begin{cases} (s^{2} - C_{+})(s^{2} - C_{-})[\partial_{x}u_{1}] = -(\alpha - 1)gH_{2}g\beta[P], \\ (s^{2} - C_{+}^{m})(s^{2} - C_{-}^{m})[\partial_{x}u_{1}] = -s(\alpha - 1)gH_{2}g\beta[\partial_{x}Q]. \end{cases}$$

For a configuration where it rains at the right side of the discontinuity, $P_-=0$ and $P_+=-Q^s\partial_x u_{1+}>0$, there exist five types of precipitation fronts:

Precipitation fronts

- 1. the dry external fronts, $\sqrt{C_+} < s < \sqrt{C_+^m}$,
- 2. the dry internal subsonic fronts, $\sqrt{C_{-}^{m}} < s < \sqrt{C_{-}}$,
- 3. the moist internal subsonic fronts, $-\sqrt{C_{-}^{m}} < s < 0$,
- 4. the moist internal supersonic fronts, $-\sqrt{C_+} < s < -\sqrt{C_-}$,
- 5. the moist external fronts, $s < -\sqrt{C_+^m}$.

This result confirms previous studies within a linear baroclinic model (Frierson *et al*, 2004).

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$$u_1(x,0) = \begin{cases} \sigma(x - x_P)^2 + U_0 & \text{if } -\sqrt{\frac{U_0}{\sigma}} \le x - x_P \le \sqrt{\frac{U_0}{\sigma}}, \\ 0 & \text{otherwise, } U_0 = 0.01, \sigma = -1 \end{cases}$$
(1)

Stationary moisture front at $x_M = 5$, saturated air at the east, unsaturated at the west:

$$Q(x,0) = Q^{s}\{1 + q_{0} \tanh(x - x_{M})H(-x + x_{M})\}, q_{0} = 0.05.$$
(2)

Strong downflow convergence in the lower layer $\rightarrow P > 0$ near the moisture front.

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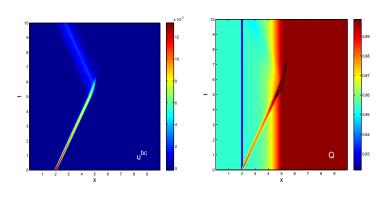
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Wave scattering on a moisture front: baroclinic velocity and moisture



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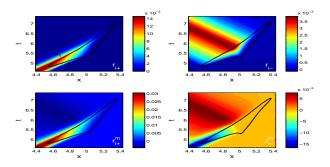
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Dry and moist internal Riemann invariants. $s_{1,2}$ -precipitation fronts (dry subsonic and moist supersonic).

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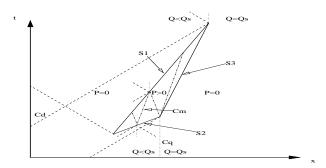
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Characteristics and fronts in the condensation zone



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In the presence of evaporation source *E*

$$\partial_t Q + \nabla \cdot (Q \mathbf{v}_1) = E - P$$

Hence:

$$\partial_t m_1 + \nabla \cdot (m_1 \mathbf{v}_1) = -\beta \mathbf{E}$$

Simple parametrizations of E (may be combined):

- ► Relaxational: $E = \frac{\hat{Q} Q}{\tau_E} H(m_1)$, where \hat{Q} equilibrium value.
- ▶ Dynamic: $E = \alpha_E |\mathbf{v}_1| H(m_1)$

 m_1 should stay positive (plays a role of static stability)

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$$ar{u}_1=0,\quad ar{\eta}_1=rac{1}{lpha-1} anh(y), \ ar{u}_2=\mathrm{sech}^2(y),\quad ar{\eta}_2=rac{-1}{lpha-1} anh(y).$$

No deviation of the free surface: $\bar{\eta}_1 + \bar{\eta}_2 = 0$. Parameters: Ro = 0.1, Bu = 10 - typical for atmospheric jets. Introduction

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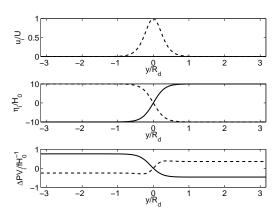
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Bickley jet: zonal velocity \bar{u}_i , thickness deviation $\bar{\eta}_i$ and PV anomaly. Lower (upper) layer: solid black (dashed gray).

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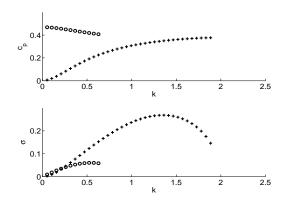
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Linear stability diagram



Phase velocity (top) and growth rate (bottom)

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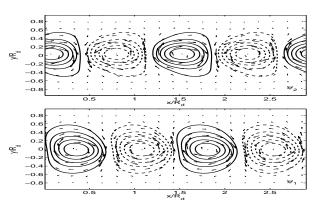
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The most unstable mode



Most unstable mode of the upper-layer Bickley jet. Upper(top) and lower (bottom) layer- geostrophic streamfunctions and velocity (arrows) fields. Large-Scale Flows 2. Modeling two-phase flows

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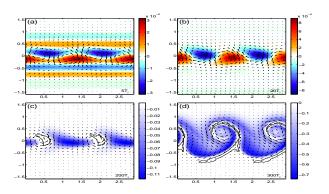
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Early stages: evolution of moisture



Evolution of the moisture anomaly $Q-Q_0$ with superimposed lower-layer velocity. Black contour: condensation zones.

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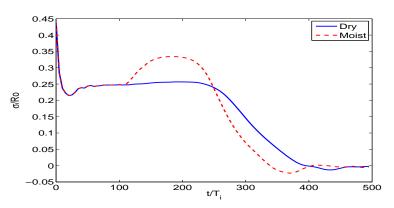
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Early stages: growth rates

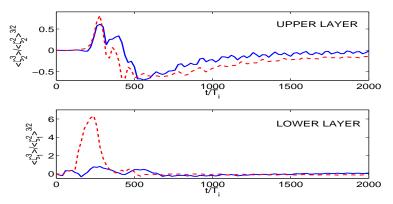


Red: moist, blue: dry simulations. \Rightarrow Transient increase in the growth rate due to condensation. Large-Scale Flows 2. Modelina two-phase flows

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Cyclone-anticyclone asymmetry



Skewness of relative vorticity. Red: moist, blue: dry simulations.

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How condensation enhances cyclones: 1-layer model

For $Ro \rightarrow$ 0 and $Bu \sim O(1)$, close to saturation $\psi \sim \tilde{q} << 1$:

$$(\partial_t + \mathbf{v}^{(0)} \cdot \nabla) \left[\nabla^2 \psi - \psi \right] = \beta P, \tag{3}$$

$$(\partial_t + \mathbf{v}^{(0)} \cdot \nabla) \left[\tilde{q} - Q_s \nabla^2 \psi \right] = -P, \tag{4}$$

 $\mathbf{v}^{(0)} = (-\partial_y \psi, \partial_x \psi)$ - geostrophic velocity, $\psi = \bar{\eta} + \eta$, and \tilde{q} is moisture anomaly with respect to Q_s .

 \Rightarrow PV of the fluid columns which pass through the precipitating regions increases. For $\tau \to 0$ $\tilde{q} \approx 0$, and:

$$Q_{\mathbf{s}}(\partial_t + \mathbf{v}^{(0)} \cdot \nabla) \left[\nabla^2 \psi \right] \approx P_{\tau \to 0} > 0,$$
 (5)

⇒ increase of geostrophic vorticity in the precipitation regions.

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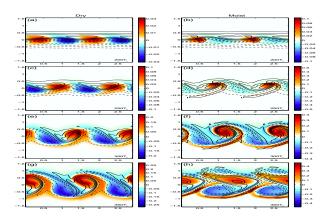
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Dry vs moist simulations: evolution of relative vorticity



Lower layer: colors, upper layer: contours. Condensation: solid black.

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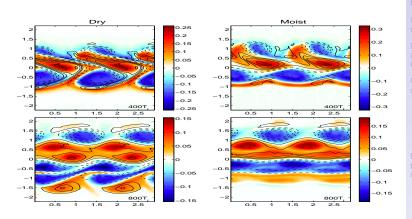
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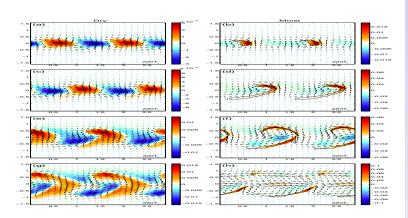
Dry vs moist simulations: formation of secondary zonal jets at late stages



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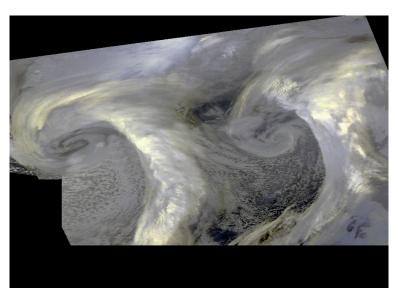
Unbalanced (aheostrophic) motions: baroclinic divergence



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Moist baroclinic instability in Nature



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The model

- Physically and mathematically consistent
- Simple, physics transparent
- Efficient high-resolution numerical schemes available
- Benchmarks: good

Moist vs dry baroclinic instability

- local enhancement of the growth rate of the moist-convective instability at the precipitation onset,
- significant increase in intensity of ageostrophic motions during the evolution of the moist instability,
- substantial cyclone anticyclone asymmetry, which develops due to the moist convection effects.
- substantial differences in the structure of zonal jets resulting at the late stage of saturation.

Conclusions



Presentation based on:

- Lambaerts J.; Lapeyre G.; Zeitlin V. and Bouchut F. "Simplified two-layer models of precipitating atmosphere and their properties" *Phys. Fluids* 23, 046603, 2011.
- Lambaerts J.; Lapeyre G. and Zeitlin V. "Moist versus Dry Baroclinic Instability in a Simplified Two-Layer Atmospheric Model with Condensation and Latent Heat Release" J. Atmos. Sci. 69, 1405-1426, 2012.

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