

#### SHIBANANDA BISWAS Indian Statistical Institute, Bangalore, India

Invariants for semi-Fredholm Hilbert module.

Coauthors: Gadadhar Misra, Mihai Putinar

For a large class of Hilbert modules over the polynomial ring, motivated by the correspondence of vector bundles with locally free sheaf, we construct a sheaf model. We see that this sheaf is coherent analytic and therefore the stalk is finitely generated at each point of the domain  $\Omega$ . Next, we obtain a (local) decomposition of the reproducing kernel in terms of these generators. An inequality involving the dimension of the joint kernel of the adjoint of the multiplication operator and the minimal number of generator of the stalk then becomes evident. In a number of examples, it is shown to be an equality. We show that there is a canonical decomposition of the reproducing kernel whenever the equality holds. This decomposition amounts to the existence of a Hermitian holomorphic vector bundle locally. We show that the equivalence class of this vector bundle is an invariant for the isomorphism class of the Hilbert module.

#### VLADIMIR BOLOTNIKOV The College of William and Mary

Functional-model realizations for contractive multipliers of the Drury-Arveson space

Coauthors: Joseph A. Ball

For operator-valued Schur-class functions (contractive-valued and analytic on the unit disc) there exist de Branges-Rovnyak functional-model coisometric, isometric and unitary realizations which under certain minimality conditions are unique up to unitary equivalence. We will present analogs of these realizations for contractive multipliers of the Drury-Arveson space. Realizations with commutative state space operators and related results concerning commutative row-contractions will be also discussed.

#### J.E. BRENNAN University of Kentucky

## Lp-bounded point evaluations for polynomials and uniform rational approximation

Coauthors: Erin Militzer

We settle an open question dating from 1973 concerning the relation that exists between approximation in the mean by polynomials and uniform rational approximation on compact subsets of the complex plane.



Lp-bounded point evaluations for polynomials and uniform rational approximation by J. E. Brennan AND E. Militzer.

Let be a nite positive Borel measure on a compact set X in the complex plane. For each p, 1 p; 1, let Hp() be the closed subspace of Lp() that is spanned by the complex analytic polynomials. It is well-known that the invariant subspace problem for subnormal operators on a Hilbert space leads directly to questions concerning approximation in H2(), and by extension to similar questions in Hp() for arbitrary p. In an initial attempt to understand the invariant subspace structure for subnormal oper- ators Wermer established in 1955 the following dichotomy valid for those measures whose closed support has zero area: Either (1) H2() has a bounded point evaluation (bpe), or (2) H2() = L2(), and his argument extends to all p, 1 p; 1. At this point the focus of attention shifted to a consideration of measures absolutely continuous with respect to area, the simplest example being dA (i.e. area measure) restricted to a compact set having no interior. Although the validity of the indicated dichotomy was subsequently conred by Thomson for all measures without exception, there nevertheless remained certain unanswered questions in the more modest context of Hp(X; dA); questions whose origins lie in operator theory. If, for example, Hp(X; dA) is assumed to have no bpe's, then it contains every rational function analytic on X. Under this assumption then, Hp(X; dA) coincides with Rp(X; dA), the closed subspace of Lp(X; dA) that is spanned by the rational functions. If, moreover, R(X) = C(X) it follows that Hp(X; dA) = Lp(X; dA). Here, C(X) is the usual space of continuous functions endowed with the uniform norm and R(X) is the closed subspace of C(X) generated by the rational functions. The question therefore arises: Does there exist a compact set X such that (1) R(X) = C(X), and (2) Hp(X; dA) = Lp(X; dA)for any, or all p, 1 p; 1? In each case the answer is no, and our verication of that fact makes use of Tolsa's theorem on the semi-additivity of analytic capacity. In the special case of a Swiss cheese X, or more generally for any set of nite perimeter, this can be established without appealing to analytic capacity and has been known since the early 1970's. This stands in striking opposition to a 1966 result of Sinanjan to the eect that: There exists a compact set X such that (1) R(X) = C(X), but (2) Rp(X; dA) = Lp(X;dA) for all p, 1 p; 1. 1 2 J. E. BRENNAN AND E. MILITZER Further, and perhaps more enlightening, examples of this sort can be constructed by by taking advantage of the fact that certain nonlinear capacities Cq, arising here by way of duality and describing the exceptional sets associated with the Sobolev space  $Wq \ 1 \ (q \neq 2)$ , decrease under a contraction, while analytic capacity does not. Department of Mathematics, University of Kentucky, Lexington, KY 40506 E-mail address: brennan@ms.uky.edu



#### MAN-DUEN CHOI University of Toronto

Joint spectral circles

Was there any useful norm estimate for A+ i B (in terms of the known spectra of two hermitian operators A and B)? Could there be any feasible way to get the sharp estimate of the norm difference of two unitary operators? In order to show sorts of magical non-commutative computations (alias, quantum computations), we look into the structure of the joint spectral circles for each pair of (non-commuting) operators on a Hilbert space. This talk also covers several papers about normal dilations as joint work with Chi-Kwong Li in the past ten years.

#### KENNETH DAVIDSON University of Waterloo

Commutant Lifting for Commuting Row Contractions

Coauthors: Trieu Le

Arveson and Muller–Vasilescu showed that every commuting *n*-tuple  $T_1, ..., T_n$  which is a row contraction has a canonical minimal dilation to an *n*-tuple of the form  $M_i^{(s)} \oplus U_i$ , where  $M_i$  are the multipliers on symmetric Fock space by the *n* coordinates functions on the complex *n*-ball, and  $U_i$  are commuting normal operators with spectrum in the unit sphere. We show that if X is a contraction commuting with each  $T_i$ , then it dilates to a contraction commuting with this dilation.

# JOERG ESCHMEIER Universitaet des Saarlandes

Multiplicities, Fredholm theory and closed range property

We indicate some applications of multiplicity theory from classical commutative algebra to multivariable Fredholmmtheory. A closed range theorem for analytic operator-valued semi-Fredholm functions is used to show that graded Hilbert space tuples are Fredholm if and only if they satisfy Bishop's property (beta) at the origin.



#### LAWRENCE FIALKOW SUNY New Paltz

Positivity of Riesz functionals and solutions of quadratic and quartic moment problems

Coauthors: Jiawang Nie, University of California at San Diego

We employ positivity of Riesz functionals to establish representing measures (or approximate representing measures) for truncated multivariable moment sequences y. We show that y lies in the closure of sequences admitting representing measures supported in a prescribed closed subset K of  $\mathbb{R}^n$  if and only if the associated Riesz functional  $L_y$  is K-positive. For a determining set K, we prove that if  $L_y$  is *strictly* K-positive, then y admits a representing measure supported in K. As a consequence, we are able to solve the truncated K-moment problem of degree k in the cases (i) (n,k) = (2,4) and  $K = \mathbb{R}^2$ ; (ii)  $n \geq 1, k = 2$ , and K is defined by one quadratic equality or inequality. In particular, these results solve the truncated moment problem in the remaining open cases of Hilbert's theorem on sums of squares.

## PALLE JORGENSEN University of Iowa

#### Representations of the Cuntz algebras, and some of their applications.

We discuss representations of the Cuntz algebras  $O_N$  in Hilbert space, and we approach them as a problem in multi-variable operator theory. While the particular families of the representation we consider are motivated by permutative representations (Bratteli-Jorgensen) and by applications to wavelets and fractals, we wish to understand these representations within a more general framework. We address the following universality question, see (1). Fix N. Display a representation of  $O_N$  on Hilbert space which is \*universal\* in the sense that every representation of  $O_N$  can be embedded in the universal one, as one of its subrepresentations. One of the tools we use is Hilbert spaces of equivalence classes of measures in the sense of Kakutani. (1) Jorgensen, Palle E. T. ; Iterated function systems, representations, and Hilbert space. Internat. J. Math. 15 (2004), no. 8, 813832.



#### MICHAEL JURY University of Florida

The commutative operator algebra of a finite-dimensional operator space

Given any finite-dimensional operator space (in other words, an *n*-dimensional subspace  $E \subset B(H)$ ), one can associate an operator algebra norm on the space of polynomials in *n* variables by declaring

$$\|p\| := \sup \|p(T)\|$$

where the supremum is taken over all *n*-tuples of commuting operators T which determine a completely contractive representation of the dual operator space  $E^*$ . For example, if Eis the column operator space  $C_n$ , then T ranges over all row contractions, and the norm is the multiplier norm of the Drury-Arveson space. If  $E = MIN(\ell_n^1)$ , then T ranges over all commuting contractions and we get the universal (or Agler) norm on the polydisk.

We prove that, as in known in the case of the above examples, the unit balls of these algerbas are described by transfer-function realizations which can be written down explicitly in terms of representations of the operator space E. The Kaijser-Varopoulos counterexample to von Neumann's inequality in the tridisk has an interesting interpretation in this context.

#### ELIAS KATSOULIS East Carolina University

#### Semicrossed products of the non-commutative disc algebras

#### Coauthors: Ken Davidson

In this talk we consider semicrossed products of the the non-commutative disc algebras by isometric automorphisms. We classify all such semicrossed products up to algebraic isomorphism, we study their representation theory and determine their C<sup>\*</sup>-envelopes.

#### MATTHEW KENNEDY University of Waterloo

#### Wandering vectors and the hyper-reflexivity of free semigroup algebras

A free semigroup algebra is the weakly closed algebra generated by n isometries with pairwise orthogonal ranges. A wandering vector is a unit vector x such that the images of x under the non-commuting words in these isometries forms an orthonormal set. We show that a free semigroup algebra either has a wandering vector, or it is a von Neumann algebra. Consequences include that every free semigroup algebra is reflexive, and that every non-self-adjoint free semigroup algebra is hyper-reflexive.



#### DEREK KITSON Trinity College Dublin

A collection of Browder joint spectra.

The classical notions of ascent and descent for a linear operator on a vector space can be extended to arbitrary collections of operators. Using this fact we construct a Browder joint spectrum for commuting n-tuples of bounded operators on a complex Banach space. This Browder joint spectrum is compact-valued and satisfies a multivariable spectral mapping theorem. In fact we arrive at a collection of Browder joint spectra with these properties and provide a new characterisation for the Taylor-Browder spectrum.

## GREG KNESE University of California, Irvine

## Rational inner functions on the bidisk

The study of rational inner functions on the bidisk differs from the study of rational inner functions on the disk (i.e. Blaschke products) in many ways. In particular, rational inner functions on the bidisk need not be regular past the boundary.

In this talk we will discuss how this irregularity on the boundary affects uniqueness of certain sums of squares decompositions of rational inner functions.

#### TERRY A. LORING University of New Mexico

Noncommutative Semialgebraic sets

Coauthors: Tatiana Shulman

We will discuss lifting rwo contractions in  $C^*$ -algebras that satisfy semialgebraic relations. For example, when C > 0 the relations

$$||xyx - yxy|| \le C$$
 and  $xx^* + yy^* \le 1$ 

are liftable out of any  $C^*$ -algebra quotient. The relations

$$xx^* - x^*x = 0$$
 and  $xx^* \le 1$ 

are famously not liftable. However, we have shown that for C > 0,

$$||xx^* - x^*x|| \le C$$
 and  $xx^* \le 1$ 

are liftable.



#### MEGHNA MITTAL University of Houston

**Operator** Algebra of Functions

Coauthors: Vern Paulsen

In this talk, we will present a theory of a special class of operator algebras, that we call "operator algebra of functions". These algebras include many important examples such as the Arveson-Drury and Schur-Agler algebras. We prove that under mild hypotheses, an operator algebra of functions is residually finite dimensional, a dual operator algebra and completely isometrically isomorphic to the multiplier algebra of a RKHS. We then use our theory to extend some of the factorization results of Ambrozie-Timotin, Ball-Bolotnikov and Kalyuzhnyi-Verbovetzkii.

## JUSTIN R. PETERS Iowa State University

Semicrossed products of Exel type

We consider semigroups of maps on a compact metric space which admit a transfer operator, and construct a semicrossed product with respect to the dynamical system.

# MIHAI PUTINAR UCSB

Extremal pluriharmonic functions in the unit ball

The still mysterious structure of the extreme rays of the cone of non-negative pluriharmonic functions in the unit ball of  $C^n$  will be discussed from the point of view of complex function theory, pluri-potential theory and multivariate operator theory. Some recent observations, derived from a joint work with Farhad Jafari, will be added.

#### SUBRATA SHYAM ROY Post doctoral fellow, Stat-math Unit, Indian Statistical Institute, 203 B. T. Road, Calcutta 700108

#### Homogeneous Operators, Jet construction and Similarity

In this paper we show, starting with the jet construction, how to construct all the irreducible homogeneous operators in the Cowen-Douglas class Bn (D) whose associated representations are multiplicity-free.



#### ORR SHALIT Technion, Israel

Representing a product system representation as a contractive semigroup and applications to dilation theory.

In this talk I will present a technical tool for analyzing representations of a product system of Hilbert C\*-modules. This tool is a construction that associates with every such representation a semigroup of contractions on a Hilbert space. Questions about the representation are translated to questions about the associated contractive semigroup. I will show how the existence of an isometric dilation for semigroups of contractions imply the existence of isometric dilations for certain representations. In particular, this leads to theorems - some known and some new - granting the existence of an isometric dilation for certain tuples of operators.

#### TAVAN TRENT The University of Alabama

On the Taylor spectrum of n-tuples of Toeplitz operators on the polydisk

In 2000 Andersson and Carlsson showed that for  $\{f_j\}_{j=1}^m \subset H^\infty(B^n)$  and their corresponding Toeplitz operators,  $\{T_{f_j}\}_{j=1}^m, \sigma_T(T_{f_1}, \ldots, T_{f_m}) = \{(f_1(z), \ldots, f_m(z)) : z \in B^n\}^{-\mathbb{C}^n},$ where  $\sigma_T$  denotes the Taylor spectrum computed with respect to the underlying Hilbert space. We show that an analogous result holds for m-tuples of analytic Toeplitz operators on the unit polydisk in  $\mathbb{C}^n$ .

# BAMDAD YAHAGHI University of Golestan, Gorgan, Iran

# An overview of triangularizability results on collections of compact operators

Let  $\mathcal{X}$  be an arbitrary Banach space. A collection  $\mathcal{C}$  of bounded operators on  $\mathcal{X}$  is called *simultaneously triangularizable* or simply *triangularizable* if there exists a maximal chain of subspaces of  $\mathcal{X}$  each of which is invariant under the collection  $\mathcal{C}$ . In this talk, I will touch upon the notion of simultaneous triangularization in both the Banach and Hilbert space settings hereby I will attempt to present a short survey of the subject.