

## DONALD CAMPBELL University of Waterloo

The Sigma-Delta Modulator as a Chaotic Nonlinear Dynamical System

Sigma-Delta modulators (or noise shapers, as they are also called) are extensively used for analogue-to-digital and digital-to-analogue data conversion (signal processing). Their dynamical behaviour can appear chaotic. I will explore this behaviour from the point of view of nonlinear dynamical systems analysis. To begin, the difference equation model of the sigma-delta modulator is introduced, and some basic results for bounded stability are obtained. The model is cast formally as a discrete dynamical system, and important continuity results allowing for a linear analysis are established. Drawing on this, I conduct a theoretical study of conditions for chaos or nonchaos using an adaptation of Devaneys definition of chaos. This study is extended to the dithered system, in the context of allowing stochatic aspects in the model. I then introduce a stochastic formulation of the long-run dynamics, which is applied to give conditions for uniformly distributed error behaviour conditions under which important consequences arise when dither is used to control the error statistics.

# OLIVER DIAZ-ESPINOSA McMaster University

# Small random perturbations of critical dynamical systems

We consider one dmensional maps that admit a renormalization group analysis. Under small random perturbations of such maps, we show that there is a universal scaling limit which is a Gaussian.

## RALUCA EFTIMIE University of Alberta

## Modeling complex spatial animal group patterns: the role of different communication mechanisms

I will present newly discovered spatial group patterns that emerge in a one-dimensional hyperbolic model for animal group formation and movement. The patterns are the result of the assumptions that the interactions governing movement depend not only on distance between conspecifics, but also on how individuals receive information about their neighbors, and the amount of information received. Some of these patterns are classical, such as stationary pulses, traveling waves, or traveling trains. However, most of the patterns are new. We call these patterns zigzag pulses, semi-zigzag pulses, traveling breathers and feathers.



## QINGWEN HU York University

Global Hopf Bifurcation of Differential Equations with State-dependent Adaptive Delay

We develop a global Hopf bifurcation theory for differential equations with state-dependent adaptive delays. This is based on an application of the  $S^1$ -equivariant degree and requires a certain  $C^1$ -smoothness of the solution operator with respect to initial value and parameters, as well as the boundedness of the inverse operator associated with the solution operator linearized in a Sobolev space consisting of periodic functions. We obtain the  $C^1$ -smoothness in the locally complete linear space  $W^{1,\infty}$  endowed with the  $|\cdot|_{W^{1,p}}$ -norm with  $1 \leq p < \infty$ , and we verify the boundedness by establishing the inverse mapping theorem and Baire's category theorem in locally complete spaces.

#### RANIS IBRAGIMOV McMaster University

# Incompressible viscous fluid flows in a thin spherical shell

Linearized stability of incompressible viscous fluid flows in a thin spherical shell is studied by using the two-dimensional Navier-Stokes equations on a sphere. The stationary flow on the sphere has two singularities (a sink and a source) at the North and South poles of the sphere. We prove analytically for the linearized Navier-Stokes equations that the stationary flow is asymptotically stable. When the spherical layer is truncated between two symmetrical rings, we study eigenvalues of the linearized equations numerically by using power series solutions and show that the stationary flow remains asymptotically stable for all Reynolds numbers. Joint with Dmitry E. Pelinovsky

# HADI JORATI University of British Columbia

# A Marcinkiewicz multiplier theorem for nonhomogeneous dilations

The classical theory of Calderon-Zygmund operators studies the class of translation invariant operators on  $\mathbb{R}^n$  that behave simply under isotropic dilations. In this work we take a specific nonhomogeneous dilation, and build the corresponding family of almostinvariant operators, where the dirac delta function has been replaced by the operator of Hilbert transform along the parabola. We characterize this class in terms of their Fourier transform, giving a Marcinkiewicz multiplier condition for nonhomogeneous dilations and establish  $L^p$  boundedness for the full range of 1 .



#### ABDOUL KANE University of Toronto

Dynamics of Synchronization in an Inhibitory Network

We consider a biophysical model describing a network of hippocampal basket cells (X. J. Wang and G. Buzski, J. Neurosci. 16:6402, 1996) and attempt to determine conditions under which certain specific modes of activity can be observed. We use the fast/slow structure of the equations involved to derive a simpler neuronal model that captures the essential features of the Wang-Buzski model interneuron and allows an analytical treatment. We then consider a pair of such neurons coupled through GABA-A type synapses and describe how the synaptic time scales interact with the intrinsic dynamics to generate various configurations (in-phase and out of phase synchrony, suppression) depending on initial conditions and parameters. We also discuss the implications of this analysis for larger networks.

#### YOUNG-HEON KIM University of Toronto

#### Curvature and the continuity of optimal transportation maps

We will discuss the continuity of optimal transport maps, in view of a semi-Riemannian structure which we have formulated recently. A necessary condition for the continuity is given as some nonnegativity condition on the curvature of this semi-Riemannian metric, and this result gives a quite general geometric frame work for the regularity theory of Ma, Trudinger, Wang and Loeper on the potential functions of optimal transport.

This is joint work with Robert McCann (University of Toronto).

# EUGENE KRITCHEVSKI McGill University

# Hierarchical Anderson Model

The hierarchical Anderson model is the random self-adjoint operator H=L+cV, where L is a hierarchical Laplacian, V is a random potential and 0 is a coupling constant measuring the strength of the disorder. In this talk, I will first review the basic properties of L and the associated spectral dimension d. Then I will present the following results about the spectral behavior of H. 1) If d<sub>i</sub>4 then, with probability one, the spectrum of H is pure point at all energies and for all c. 2) If d<sub>i</sub>1 then, in a natural scaling limit, the eigenvalues of finite volume approximations to H converge to a Poisson point process.



## PAUL LEE University of Toronto

#### A Nonholonomic Moser Lemma

In the 70's, Moser showed that given two volume forms with the same total volume on a compact orientable manifold, there exists a diffeomorphism which pushes one to another. In this talk, I will describe the generalization of this to the case of a manifold with a bracket generating distribution. If time permits, I will also talk about the infinite-dimensional geometry associated with it. It turns out that all these can be considered as the path lifting property of a certain infinite-dimensional principle bundle with the Carnot-Caratheodory geometry. This is joint work with Boris Khesin.

#### VLADISLAV PANFEROV McMaster University

#### On a kinetic averaging lemma

I will review the ideas behind the type of results known as averaging lemmas and indicate some possible applications to the regularity of solutions of the 1D nonlinear Boltzmann equations.

## MARY PUGH University of Toronto

Notes on Blowup and Long-Wave Unstable Thin Film Equations

I will survey the subcritical, critical, and supercritical regimes of the long-wave unstable thin film equation  $u_t = -(u^n u_x x x)_x - (u^m u_x)_x$  on the line, subject to nonnegative initial data  $u_0 \ge 0$ . I will discuss how different methodologies could have been used to guess at these regimes as well as what types of results have been found. This will be a pedagogical talk and should, I hope, be accessible to all. This is joint work with Andrea Bertozzi (UCLA), Richard Laugesen (UIUC), and Dejan Slepcev (CMU).



## C. EUGENE WAYNE Boston University

Stability of Vortex Solutions of the Two and Three Dimensional Navier-Stokes Equations

Vortex solutions play an important role in organizing the behavior of viscous fluid flows. Indeed they have been referred to asthe "sinews" of turbulence. In this talk I will explain how using ideas from dynamical systems theory and kinetic theory one can understand the stability of certain vortices in both the two and three dimensional Navier-Stokes equations.

#### IGOR WIGMAN McGill University

# Nodal sets for random eigenfunctions of the Laplacian on the torus

We study nodal sets for typical eigenfunctions of the Laplacian on the standard torus in 2 or more dimensions. Making use of the multiplicities in the spectrum of the Laplacian, we put a Gaussian measure on the eigenspaces and use it to average over the eigenspace. We consider a sequence of eigenvalues with multiplicity N tending to infinity. The quantity that we study is the Leray, or microcanonical, measure of the nodal set. We show that the expected value of the Leray measure of an eigenfunction is constant. Our main result is that the variance of Leray measure is asymptotically 1/(4 pi N), as N tends to infinity, at least in dimensions 2 and at least 5.

# ALBERTO MONTERO ZARATE University of Toronto

## On a weighted Hodge decomposition problem that arises in the study of the Gross-Pitaevskii energy

In this talk I will discuss a Hodge decomposition problem in a bounded domain in 3-d, with respect to a weight function that vanishes linearly on the boundary. This problem arises in the study of the Gross-Pitaevskii (GP) energy, and some of its particular solutions are known. We find that much of the classical theory of Hodge decompositions can be applied to our situation once one solves a specific degenerate elliptic problem (degenerate on the boundary, because of the weight). As an application, one can show that a Gamma-convergence result for the GP energy, known in 3-d when the data of this functional leads to the known particular solutions of the Hodge decomposition problem mentioned above, holds for more general data as well.