FOR RESEARCH IN MATHEMATICAL SCIENCES

ABSTRACTS 1.2

PAVEL BLEHER IUPUI

Exact Solution of the Six-Vertex Model with Domain Wall Boundary Condition

The six-vertex model, also known as the square ice model, with domain wall boundary condition (DWBC) has been introduced and solved for finite N by Korepin and Isergin. The solution is based on the Yang-Baxter equations and it represents the free energy in terms of an NxN Hankel determinant. Paul Zinn-Justin observed that the Isergin-Korepin formula can be re-expressed in terms of the partition function of a random matrix model with a nonpolynomial interaction. We use this observation to obtain the large N asymptotics of the six-vertex model with DWBC in the disordered phase. The solution is based on the Riemann-Hilbert approach and the Deift-Zhou nonlinear steepest descent method. As was noticed by Kuperberg, the problem of enumeration of alternating sign matrices (the ASM problem) is a special case of the the six-vertex model. We compare the obtained exact solution of the six-vertex model with known exact results for the 1, 2, and 3 enumerations of ASMs, and also with the exact solution on the so-called free fermion line.

DAVID BRYDGES UBC

The Renormalisation Group and Self Avoiding Walk

The self-avoiding walk on the simple cubic lattice in four dimensions is conjectured to have an end-to-end distance that grows proportionally to $\sqrt{N} \ln^{1/8} N$, where N is the number of steps in the walk. I will review a program for proving this and describe what has been accomplished so far.

MICHAEL FISHER Maryland

Exploring critical singularities via spherical models: Beguiling but wayward

JOHN GRACEY Liverpool

Practicalities of renormalizing quantum field theories

The talk will review basic ideas of the renormalization of a quantum field theory before discussing practical methods of extracting the renormalization group functions to several orders in perturbation theory. Recent results concerning the renormalization of quantum chromodynamics in various gauges and other field theories are used to illustrate the technical issues.

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LI GUO Rutgers

Rota-Baxter algebra method in the Hopf algebra approach of pQFTrenormalization

We discuss the role played by the Rota-Baxter operator in the Hopf algebra approach of Connes and Kreimer to the renormalization of perturbative quantum field theory. In their work, Feynman graphs are organized into a Hopf algebra and a regularized Feynman rule is given by an algebra homomorphism from this Hopf algebra to a Rota-Baxter algebra. Built on classical results in Rota-Baxter algebras such as Altkinson's factorizations and Spitzer's identity, we explain how pQFT results such as the algebraic Birkhoff decomposition for renormalization and the Bogoliubov formula can be derived from theorems on Rota-Baxter algebras. We further show that the Feynman rules have a matrix representation that converts the process of renormalization to matrix calculations. Related rooted trees and decompositions will also be discussed.

STEFAN HOLLANDS Gottingen

Quantum fields in curved space

During the past 5-10 years, there has been considerable progress in the construction of renormalized, perturbative, interacting quantum field theories on general curved spacetime manifolds. This progress has become possible with the help of mathematical methods from "Microlocal Analysis" on the one hand, and a new conceptual viewpoint on perturbative QFT emphasizing the algebraic and "local and covariant" nature of quantum fields. The mathematical methods were important in order to control the singularities of field products, and the algebraic viewpoint was essential to formulate the theory without using a preferred "vacuum state", which is not available in a general curved space. In my talk, I want to present the key ideas in the subject and outline its present status.

VLADIMIR KOREPIN YITP, Stony Brook

Universality of Entropy Scaling in One-Dimensional Gap-less Models

Quantum models with unique ground state are considered. Von Neumann entropy of a part of the ground state [subsystem] is calculated. It shows the entanglement of this part with the rest of the ground state. For a large subsystem asymptotic behavior of the entropy can be related to the gap in the spectrum of the Hamiltonian. For gap-less models the entropy increases logarithmically, for gap-full models it saturates. Several different models will be considered: spin chains, Bose gas, Hubbard model.

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DIRK KREIMER CNRS-IHES and Boston

Renormalization: from Hopf algebras to number theory

The talk will explain how a recursive procedure like the Bogoliubov recursion leads to a Hopf algebra, and how notions like self-similarity appear naturally in that algebraic context. It then continues to explore the polylogarithm and its algebraic structure which are very similar, and reviews the manifold connections between number theory and quantum field theory which have emerged in recent years.

R. S. MACKAY Warwick

Quantum statistical mechanics of Frenkel-Kontorova chains

We extend a renormalisation operator for the transition by breaking of analyticity of incommensurate minimum energy states of Frenkel-Kontorova chains to one for the quantum statistical mechanics. The known fixed point for the former operator induces two for the latter, both at temperature zero, with $\beta\hbar = 0$ and ∞ , respectively. Scaling exponents are deduced from the those for the former case. This is joint work with N.Catarino.

GERRY MCKEON UWO

Using the Renormalization Group

Using the fact that the renormalization scale parameter is arbitrary, and by resumming perturbative expansions for physical quantities, a number of calculations are outlined. Among them are a summing of poles in the expansion of the bare coupling in terms of the renormalized coupling, the leading log contributions to the effective action in the presence of an instanton, the effective action in N = 2 super Yang-Mills theory, and the effective potential in massless theories involving scalars.

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TIM R. MORRIS Southampton

Manifestly gauge invariant exact renormalization group

We construct a manifestly gauge invariant Exact Renormalization Group for SU(N) Yang-Mills theory, in a form suitable for calculations without gauge fixing at any order of perturbation theory. The effective cutoff is incorporated via a manifestly realised spontaneously broken SU(N|N) gauge invariance. Diagrammatic methods are developed which allow the calculations to proceed without specifying the precise form of the cutoff structure. We confirm consistency by computing for the first time both the one and two loop beta function coefficients without fixing the gauge or specifying the details of the cutoff. We conclude by sketching how quarks fit in and a new non-perturbative approximation scheme.

JOHN PALMER Arizona

Short distance behavior of the subcritical scaling functions for the 2d Ising model

Mass zero limits for the Green function of a Dirac operator are used to determine the short distance behavior of the scaling functions for the two dimensional Ising model. The Sato, Miwa and Jimbo characterization of the log derivative of the scaling function plays a crucial role.

TOM SPENCER IAS

Supersymmetric models of random matrices and convexity

This talk will describe a supersymmetric approach to band random matrices. Convexity plays an important role in the analysis the hyperbolic sigma model arising from the Bosonic sector of this supersymmetric model. I shall also offer some speculative comments about how to combine convexity and renormalization group methods.

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ABSTRACTS 1.2

IVAN TODOROV Bulgarian Academy of Sciences

The role of conformal invariance in quantum field theory

Dilation and conformal invariance in quantum field theory require the existence of a renormalization group fixed point and exploit partial summation of the perturbation series. I an early work of G. Mack and the speaker a skeleton expansion is constructed (in terms of conformally invariant dressed propagators and vertex functions) that is free of ultraviolet divergences in a certain range of anomalous dimensions. In a second part of the lecture the (stronger) condition of global conformal invariance (GCI) is explored that yields rational correlation functions and a higher (in particular D = 4) extension of the notion of vertex algebra. Copnformal partial wave expansions are used in a recent work of N. Nikolov, K.-H. Rehren and the author to establish Wightman positivity at the 4-point level.

STEFAN WEINZIERL Mainz

The Art of Computing Loop Integrals

Loop integrals in quantum field theory may contain ultraviolet and infrared divergences. After the introduction of a regulator, like dimensional regularization, they are computed as a Laurent series in the regularization parameter. I will discuss systematic algorithms for the evaluation of Feynman integrals. These algorithms are based on Hopf algebras and evaluate the Feynman integral to (multiple) polylogarithms.

JEAN ZINN-JUSTIN CEA/Saclay

The critical temperature of the weakly interacting Bose gas