FOR RESEARCH IN MATHEMATICAL SCIENCES

## ERIC BEDFORD Indiana

On the cohomological approach to birational dynamics

A rational mapping  $f: C^k - - - > C^k$  is said to be birational if it has a rational inverse. We will discuss some aspects of the dynamics of birational maps. In particular, we address the question of how to determine the dynamical degree, which is the growth rate of degree  $(f^n)$  with respect to n. We give some examples of mappings where it is possible to determine the dynamical degree by studying the action of  $f^n$  on the cohomology group  $H^{1,1}$ . These examples exhibit behaviors that do not occur in dimension 2. This is joint work with Jeff Diller and with Kyounghee Kim.

## ARACELI BONIFANT Rhode Island

#### Intermingled Basins

We will talk about a family of rational maps of  $P^2(C)$  or  $P^2(R)$  that leaves invariant an elliptic curve. For some values of the parameters, the elliptic curve is an attractor with riddled basins dense, in the Julia set. In fact for some values of the parameters there is a line that is also an attractor with basin dense in the same Julia set.

#### BODIL BRANNER Technical University of Denmark

#### A Holomorphic Tale

Glimpses of Holomorphic Dynamics in light of John Milnor's influence on its development

## XAVIER BUFF Paul Sabatier

Julia Sets of Positive Measure IV: Upper semi-continuity of Siegel disks

Let  $\mathcal{S}$  be the set of rational or irrational numbers whose continued fractions have coefficients greater than the Inou-Shishikura constant. Let  $\widetilde{\mathcal{S}}$  be the subset of those which are of bounded type.

I will show that if  $\theta_0 \in \widetilde{S}$ , as  $\theta$  tends to  $\theta_0$  within S, the limit superior of the postcritical set of  $P_{\theta} : z \mapsto e^{2i\pi\theta}z + z^2$  is contained in the closure of the Siegel disk of  $P_{\theta_0}$ .

ABSTRACTS 1.2

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## ARNAUD CHERITAT Paul Sabatier

## Julia Sets of Positive Measure II : preservation of half the area of Siegel disc

Given a quadratic polynomial P with a fixed Siegel disk D, given a P- invariant subdisk  $D_0$ , and a scale  $\epsilon$ , we provide special perturbations of the parameter such that the new quadratic polynomial P' has a Siegel disk D' with the following properties: - The rotation number of D' is close to that of D - D' contains  $D_0 - P'$  has a cycle Hausdorff-close to the boudary of  $D_0 - D'$  roughly covers more than half the area of all balls of radius  $i \epsilon$  contained in D

If the rotation number of D has all its continued fraction entries greater than M, then the same holds for our perturbations D'.

## LAURA DE MARCO Chicago

## Polynomial dynamics and trees

The dynamics of a polynomial on its basin of infinity is modelled by a tree with dynamics. In analogy with spaces of hyperbolic structures, the moduli space of polynomials can be compactified by a space of trees. I'll discuss basic properties of these trees and some applications to the behavior at infinity of unbounded families of polynomials. This is joint work with C. McMullen.

## BERTRAND DEROIN IHES

Some properties of the holonomy pseudo-group of a holomorphic foliation

We will discuss some properties of the holonomy maps (i.e. Poincaré map) of a singular holomorphic foliation of a complex surface.

## ROBERT L. DEVANEY Boston

### Parameter Plane Structures for Singularly Perturbed Rational Maps

In this talk we describe some of the different types of structures found in the parameter planes for families of rational maps of the form  $z^n + C/z^d$ . These structures include fractal snowflakes and Cantor necklaces, rings around the McMullen domain, Cantor sets of circles of Sierpinski curve Julia sets, and "external" baby Mandelbrot sets.

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## ADRIEN DOUADY Ecole Normale Superieure

Julia Sets of Positive Measure I : Overview

We describe the context of the question and remind some well known properties of  $f_{\theta} = e^{2i\pi\theta}z + z^2$ . Then we list the ingredients of the proof by A. Cheritat and X. Buff of the existence of a  $\theta$  Cremer with  $m(K_{\theta}) > 0$ : Mc Mullen's density theorem; preservation of half the area of the Siegel disc (JSPM 3); upper semi continuity of the Siegel disc (JSPM 2), which relies on a renormalization property (JSPM 1). Finally we give a brief sketch of the final proof, which shall be developed in JSPM 4.

Julia Sets of Positive Measure V: The bouncing

Fix  $\theta_0 = [a_1, a_2, ...]$  (continued fraction) of bounded type with all coefficients  $a_i$  greater than the Shishikura-Inou constant S. Choosing n and N great enough (in this order), take  $\theta = [a_1, ..., a_n, N, S, S, S, ...]$ . Set  $f_{\theta}(z) = e^{2i\pi\theta}z + z^2$ , denote by  $K_{\theta}$  and  $\Delta_{\theta}$  its filled Julia set and its Siegel disc; define in the same way  $f_0$ ,  $K_0$  and  $\Delta_0$ .

Choose  $\delta$  very small and define the annulus  $W = \{z \in \mathbf{C} | 2\delta < d(z, \Delta_0) < 10\delta\}$ , so that a point in  $\Delta_0$  cannot escape to infinity under  $f_{\theta}$  without stepping in W. By JSPM II, we can suppose that  $(\forall z \in \Delta_{\theta})d(z, \Delta_0) < \frac{1}{2}\delta$ . Denote by  $X_p$  the set of points in  $\Delta_0$  which bounce at least p times between  $\Delta_0$  and W under  $f_{\theta}$ , and set  $X_p^* = X_p - X_{p+1}$ .

Using Mc Mullen's density theorem, Koebe distortion estimates and JSPM III, we get estimates on the measure of  $X_p$  and  $X_p^*$ , and finally show that  $m(\Delta_0 \setminus K_\theta) = \sum m(X_{2p+1}^*)$ can be made arbitrarily small. From this the construction of a  $\tau$  such that  $K_{\tau}$  has empty interior but positive measure follows easily.

### ROMAIN DUJARDIN Universite Paris 7

#### Currents and measures in parameter space

Let  $\{f_{\lambda}\}_{\{\lambda \in \Lambda\}}$  be any algebraic family of rational maps of a fixed degree, with a marked critical point  $c(\lambda)$ . We first prove that the hypersurfaces of the parameter space  $\Lambda$  on which  $c(\lambda)$  is periodic converge as a sequence of positive closed (1, 1) currents to the bifurcation current attached to c and defined by DeMarco.

We then turn our attention to the parameter space of polynomials of a fixed degree d, with all critical points marked. By intersecting the d-1 currents attached to the critical points, we obtain a positive measure  $\mu_{\text{bif}}$  of finite mass, supported on the connectedness locus (which was already studied by Bassanelli and Berteloot). We give several characterizations of this measure, already well known in the unicritical case. In particular we show that its support is precisely the closure of the set of strictly critically finite polynomials (i.e. of Misiurewicz points).

This is joint work with Charles Favre.

ABSTRACTS 1.2

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## ADAM EPSTEIN Warwick

## A Bit of Abstract Nonsense

We discuss a categorical construction of, and comparison of, the direct and inverse limits of a system of Banach spaces which is simultanously, and compatibly, directed "upwards" by isometric inclusions, and "downwards" by projections. This exercise in pure functional analysis provides a foundation for the infinitesimal deformation theory of inverse limit Riemann solenoids. Examination of the duality encountered reveals that the notion of "cotangent space to Teichmuller Space" is somewhat subtle.

## JOHN HUBBARD Cornell

tba

## JEREMY KAHN Stony Brook

The complex a priori bounds for bounded primitive renormalization

Let  $f(z) = z^2 + c$  be infinitely renormalizable of *bounded primitive* type; it admits an infinite sequence of primitive renormalizations, and the period of each is a bounded multiple of the next-smaller period. We show by means of a combinatorial approximation of the geometry that "if it is bad now, it was worse earlier": if some primitive renormalization can be performed only with small modulus, then there is a previous level that can be performed only with smaller modulus. It follows immediately that all renormalizations can be performed with modulus bounded below.

TAN LEI Cergy-Pontoise

Constructing rational dynamics using Thurston's theorem

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ABSTRACTS 1.2

## VOLODYMYR NEKRASHEVICH Texas A and M

Combinatorial equivalence of topological polynomials and group theory

We will describe a method of determining when two Thurston maps (post-critically finite topological polynomials) are combinatorially equivalent. In particular, we will answer questions posed by J. Hubbard and A. Douady about compositions of Dehn twists with  $z^2 + i$  and with the "rabbit". On our way we will get many exotic groups: an uncountable family of "indistinguishable" groups, groups of intermediate growth, exotic amenable groups, etc.

### LEX OVERSTEEGEN University of Alabama at Birmingham

## On Cremer Julia sets

We study the topology of the Julia J set of a quadratic Cremer polynomial P. It is known that such a Julia set is a non-locally connected, non-separating one dimensional plane continuum. We show that if there exists an external angle whose impression does not contain the fixed Cremer point p, then J is connected im kleinen at a dense set of points and these points are contained in a unique, degenerate impression. (authors-A.Blokh and L.Oversteegen)

## DIERK SCHLEICHER Bremen

Escaping Points of Entire Functions: Proofs and Counterexamples to Questions of Fatou and Eremenko

Many of the deepest results in polynomial dynamics are obtained using dynamic rays and their landing properties. The rays are constructed using the simple form of the dynamics near infinity. For transcendental entire functions, infinity is an essential singularity without simple normal form.

Fatou and Eremenko asked whether points converging to infinity ("escaping points") have the form of curves to infinity; escaping points are always contained in the Julia set. We show that this true for large classes of bounded type entire functions, including those of finite order. We also show that there is a bounded type entire function for which every path component of the Julia set, and especially of the set of escaping points, is bounded.

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ABSTRACTS 1.2

## MITSUHIRO SHISHIKURA Kyoto

Julia Sets of Positive Measure III: the parabolic renormalization

For a holomorphic map f with a parabolic fixed point with derivative 1 and non-zero second derivative, the attracting and repelling Fatou coordinates are defined so that they conjugate the dynamics to the translation  $z \mapsto z + 1$  on half-neighborhoods of the fixed points, on attracting and repelling sides respectively. The horn map is the correspondence between the two Fatou coordinates induced by the orbits going from the repelling side to the attracting side. Via the quotient by  $z \mapsto z+1$  and the exponential map  $z \mapsto e^{2\pi i z}$ , and after a suitable normalization, the horn map defines a holomorphic map g near the origin with derivative 1. This construction defines the *parabolic renormalization*, which plays a crucial role in the study of perturbed map  $z \mapsto e^{2\pi i \alpha} z + \ldots$ , where  $\alpha$  is an irrational number whose continued fraction coefficients are large. In this talk, we will present our result (joint with Hiroyuki Inou) which asserts that there exists a concrete space of holomorphic maps invariant under the parabolic renormalization and persistent under a perturbation. We also obtain the result that the near-parabolic or cylinder renormalization is hyperboloic on the space of maps with small rotation angles at the fixed point.

## NESSIM SIBONY Paris Sud

### The ddc method in dynamics of several complex variables

We review some recent results on the dynamics of holomorphic maps (or even meromorphic transforms ) in several complex variables. We focus on the construction of measures of maximal entropy and their statistical properties (mixing, exponential decay of correlations). We will explain the method in two technically simple cases: polynomial like mappings and horizontal like mappings. The basic tools are: appropriate spaces of test forms adapted to complex dynamics, resolution of the ddc equation with good estimates, intersection of positive closed currents of any bidegree. This is joint work with T.C.Dinh