THE FIELDS INSTITUTE

ABSTRACTS 1.2

FOR RESEARCH IN MATHEMATICAL SCIENCES

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Multiple flag varieties of tame type: The s-tame dimension vectors of stars (25-30)

A quiver $Q = (Q_0, Q_1)$ is given by a set of vertices Q_0 and a set of arrows Q_1 and two maps $s, t : Q_1 \to Q_0$ which assign to every arrow $\alpha \in Q_1$ its starting point $s(\alpha)$ and its terminating point $t(\alpha)$. A star Q is a quiver with no multiple arrows and no oriented cycles, with a unique sink which is the only possible branching vertex (thus, Q is obtained from linearly oriented quivers of type \mathbb{A} by identifying the sinks).

A subspace representation of a star $Q = (Q_0, Q_1)$ is a collection of vector spaces V_i , $i \in Q_0$, over a particular field K together with K-linear maps $V_{\alpha} : V_{s(\alpha)} \to V_{t(\alpha)}, \alpha \in Q_1$, which are all injective.

In 1999, the classification of all dimension vectors for stars, which occur for only finitely many isomorphism classes of subspace representations, was given by P. Magyar, J. Weyman, and A. Zelevinsky (see [MWZ]). In this way, Magyar, Weyman, and Zelevinsky deal with multiple flag varieties of finite type. Multiple flag varieties are part of the Schubert calculus for reductive groups. Here one takes a reductive group G and studies the G-orbits and their closures in the product $G/P \times G/Q$, where P and Q are parabolic subgroups in G. In this case the number of orbits is always finite. But if one generalises this to the action of G on $G/P_1 \times \cdots \times G/P_k$ with a tuple (P_1, \ldots, P_k) of parabolic subgroups, this is no longer true. What happens in the case $G = GL_n$ is shown in [MWZ]. The aim of this talk is to present the list of all socalled "s-tame" dimension vectors of stars. These are those dimension vectors for which there exists at least one one-parameter family of subspace representations, but no (not necessarily indecomposable) n-parameter family of subspace representations with $n \geq 2$. In this way, the problem of multiple flag varieties of tame type is solved. I am going to talk about the methods of finding the s-tame dimension vectors. Furthermore, I would like to show some typical phenomena which occur when constructing the indecomposable families explicitly.

References

P. Magyar, J. Weyman, A. Zelevinsky, Multiple Flag Varieties of Finite Type, Advances in Mathematics 141, pp. 97–118 (1999)